As 18, the number of quotients in a complete period, is even, therefore $x = p_{18}$ and $y = q_{18}$.

$\frac{p_1}{q_1} = \frac{75}{1}$, $\frac{p_2}{q_2} = \frac{301}{4}$, $\frac{p_3}{q_3} = \frac{376}{5}$, $\frac{p_4}{q_4} = \frac{677}{9}$, $\frac{p_5}{q_5} = \frac{3084}{41}$, $\frac{p_6}{q_6} = \frac{9929}{132}$, $\frac{p_7}{q_7} = \frac{22942}{305}$, $\frac{p_8}{q_8} = \frac{32871}{437}$, $\frac{p_9}{q_9} = \frac{121555}{1616}$, $\frac{p_{10}}{q_{10}} = \frac{762201}{10133}$, $\frac{p_{11}}{q_{11}} = \frac{2408158}{32015}$, $\frac{p_{12}}{q_{12}} = \frac{3170359}{42148}$, $\frac{p_{13}}{q_{13}} = \frac{8748876}{116311}$, $\frac{p_{14}}{q_{14}} = \frac{29416987}{391081}$, $\frac{p_{15}}{q_{15}} = \frac{126416824}{1680635}$, $\frac{p_{16}}{q_{16}} = \frac{155833811}{2071716}$, $\frac{p_{17}}{q_{17}} = \frac{282250635}{3752351}$, $\frac{p_{18}}{q_{18}} = \frac{1284836351}{17081120}$.

$$\therefore \qquad x = 1284836351, \qquad y = 17081120.$$

I have verified these numbers.

NOTE ON THE SOLUTION OF MR. HOLBROOK'S QUESTION.

BY THE EDITOR.

Prof. Eddy (see p. 126) has given the equation of a solid whose surface is generated as described in the question proposed by Mr. Holbrook on page 72; but it appears from a subsequent clause that Mr. Holbrook had a different question in view, viz.; he asks for a "demonstration that no two ellipses can be parallel."

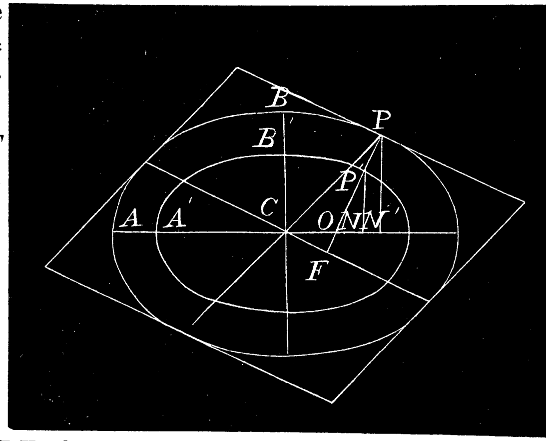
The surface of a solid with an elliptical base, horizontal sections of which shall be bounded by curves parallel to the periphery of the base, may be generated by a straight line which makes a constant angle with the *normal* to the ellipse while the extremity of the line describes the periphery of the ellipse. That no horizontal section, above the base of such a solid, can be an ellipse, is what we understand Mr. Holbrook to assert and desire to have demonstrated.

Horizontal sections of the solid of which Prof. Eddy has given the eqnation, are obviously ellipses; but that no section, above the base, of the solid whose surface is generated as above described, can be an ellipse, may be demonstrated as follows:

Let ABP (see diagram on next page) represent an ellipse whose semi-axes are $AC = a$ and $BC = b$, and the normal of which, at any point P, is $PO = N$. Let $A'B'P'$ be a parallel curve within the ellipse; then will the portion PP' of the normal, be of the same length for all points of the ellipse. We may therefore put $PP' = c$, a constant.

By property 12 of the ellipse, (see Bridge's Conic Sections), we have $PO \times PF = BC^2$. And, assuming the parallel curve $A'B'P'$ to be an ellipse, we have, by the same property, $P'O \times P'F = B'C^2$. By substituting for $P'O$, $P'F$ and $B'C$ their equivalents $PO - c$, $PF - c$, and $BC - c$, we get $PO + PF = 2BC$, from which, by squaring both sides we get $PO^2 + 2PO \times PF + PF^2 = 4BC^2$; but $2PO \times PF = 2BC^2$; $\therefore$ by subtraction we get

$$PO^2 + PF^2 = 2BC^2.$$



Because $2BC^2 = 2PO \times PF$, therefore $PO^2 + PF^2 = 2PO \times PF$. But this equation can only be true when $PO = PF$, and in that case the curve ABP is a circle. Hence our assumption that the parallel curve is an ellipse, is not true; and, therefore, no curve that is parallel to an ellipse can be an ellipse.

The equation of the parallel curve may be found as follows: Let x, y be rectangular coordinates to any point P of the ellipse, the origin being at the center, and N the normal at the point P. Also, let x', y' be rectangular coordinates and N' the normal to the corresponding point P' of the parallel curve $A'B'P'$; and let a, b be the semi-axes of the ellipse and a', b' the semi-axes of the parallel curve, then is

$$a = a' + c,\ b = b' + c, \text{ and } N = N' + c, \ldots\ldots \quad (1)$$

and from similar triangles we find $y = \dfrac{N' + c}{N'} y'. \ldots\ldots\ldots\ldots \quad (2)$

The equation for the length of the normal is readily found to be

$$N = y\sqrt{\left(1 + \frac{dy^2}{dx^2}\right)}. \ldots\ldots\ldots\ldots \quad (3)$$

Differentiating the equation to the ellipse, we find

$$\frac{dy^2}{dx^2} = \frac{b^4 - b^2y^2}{a^2y^2}.$$

Therefore, by substitution, $N = \dfrac{1}{a}\sqrt{\left[(a^2 - b^2)y^2 + b^4\right]}. \ldots\ldots\ldots \quad (4)$

Substituting in (4) from (1) and (2) we get, for the required equation,

$$y' = \frac{N'}{N' + c}\sqrt{\left[\frac{[(N' + c)(a' + c)]^2 - (b' + c)^4}{(a' + c)^2 - (b' + c)^2}\right]}.$$